Arbitrary Descriptive Names and the Benacerraf Problem

The paper explores the question of reference to natural numbers by focusing on the tension between two claims: the *singularity claim*, which roughly says that sentences like "3 is *G*" express singular propositions where "3" refers to a single abstract object; and the *arbitrariness claim*, which says roughly that it is arbitrary which object "3" refers to. The arbitrariness claim plays a crucial role Paul Benacerraf's highly influential argument that, from the existence of multiple and equally adequate set theoretical reductions of natural numbers to sets, concludes that numbers don't exist.
The paper argues for the compatibility between the singularity claim, the arbitrariness claim and the existence of numbers on the grounds that numerals are arbitrary descriptive names.

1 The Benacerraf Problem

In his 1965 seminal paper "What Numbers Could Not Be", Paul Benacerraf establishes a constraint that any account of what numbers are should meet in order to be arithmetically adequate. The constraint has it that any account should entail the truth of the theorems of arithmetic in the standard "at face value" semantics, and explain cardinality in terms of counting. Call this the *formal adequacy constraint*. Then, Benacerraf contends that (infinitely) many sequences of objects satisfy such a constraint, e.g. the Zermelo ω-sequence and the Von Neumann ω-sequence. On these grounds, Benacerraf argues that there is no non-arbitrary reason in favour of the thesis that there is a unique sequence of objects which is the natural numbers. From this arbitrariness claim, Benacerraf concludes that arithmetical realism, viz. the view that natural numbers exist, is false.

2 Numerals as arbitrary descriptive names

My aim is to articulate an as yet unexplored response to the Benacerraf Problem which allows us to keep arithmetical realism, the arbitrariness claim and the singularity claim together.
I deem useful to start out by reflecting on how the reference of numerals is fixed. Apparently, numerals are proper names. And yet, the usual causal-historical theory of reference of ordinary proper names which maintains that reference is fixed by a reference-fixer who is acquainted with a given object *o* and baptises it doesn't directly carry over to the case of numerals. For their referents, i.e. abstract objects, are amongst the classical examples of objects with which we cannot be acquainted. So, what kind of expressions are numerals? In this paper, I will defend the contention that numerals are *arbitrary descriptive names*.
Descriptive names get their reference fixed via a uniquely reference-fixing definite description, used attributively, which picks out the object the name refers to. A famous example is that of Leverrier,[1] who in 1845 and prior to any telescopic confirmation, coined the name "Neptune" to refer to the (then unknown) planet responsible for the perturbations in the orbit of Uranus.
Similarly, I submit that the reference of numerals is fixed via a uniquely reference-fixing definite description. An arbitrary numeral "n" refers to whichever object uniquely satisfies the definite description "the *n*th satisfier of the formal adequacy constraint". So, "0" refers to whichever object uniquely satisfies the description "the satisfier of the formal adequacy constraint which has no predecessor"; "1" refers to whichever object uniquely satisfies the description "the satisfier of the formal adequacy constraint which is the successor of 0", and so on and so forth.
According to a Millian account of descriptive names,[2] the semantic content of a descriptive name is the denotation (if any) of the description that fixes its reference. To put it differently, the definite description is used to fix the name's content but it is not part of the content of the name. Therefore, descriptive names are devices of direct reference, for the content of a directly referring expression is nothing more than its referent. In light of this, we can maintain that an assertive utterance of the

[1] The example is Kripke's.
[2] See e.g. Jeshion (2004), Kripke (1980).

atomic sentence “3 is *G*” expresses a singular proposition: *x is G*, where *x* is 3 itself. This enables us to preserve the singularity claim.

In order to take up the Benacerraf Problem, it is important to understand what the arbitrariness claim really amounts to. Following the lead of Boccuni (2013), Breckenridge and Magidor (2012), Carrara and Martino (2010), I take the notion of arbitrariness to be *epistemic* in kind: roughly put, even though “n” picks out a single object, we don’t know which object “n” refers to. More precisely: there are (infinitely) many sequences of entities satisfying the formal adequacy constraint, but we lack the epistemic access to the realm of abstract objects needed to know which particular sequence the natural numbers are.

The idea that numerals are descriptive names is absolutely compatible with the epistemic interpretation of the arbitrariness claim. Consider all entities that are able to satisfy the formal adequacy constraint. Of all of them, the object that the term “n” picks out is fixed via the description “the *n*th satisfier of the formal adequacy constraint”. So, “n” refers to whichever object uniquely satisfies this description. Thus, numerals can be regarded as arbitrary descriptive names.

3 A Response to the Benacerraf Problem

On the view I’m recommending, although the supporter of Benacerraf’s argument is right in holding that it is arbitrary, i.e. we don’t know, which object “0” refers to, “0” refers to whichever object uniquely satisfies the description “the satisfier of the formal adequacy constraint which has no predecessor”. Thus, the response to the Benacerraf Problem is that epistemic matters should be distinguished from semantics (and metaphysical) matters: in order for a numeral to refer to an object, we need not know its actual referent. Our epistemic access to the sequence of objects which is the natural numbers is not a necessary condition for its existence. Therefore, it is possible to claim that there is a unique sequence of objects which is the natural numbers, while nonetheless admitting that we don’t know which it is.

The picture just sketched out is in line with a non-eliminative version of arithmetical structuralism. As far as the reference of numerals is concerned, a non-eliminative version of structuralism is certainly compatible with the idea that numerals are arbitrary descriptive names, for all that matters in the structuralist perspective is that numerals refer to whichever objects satisfy the formal adequacy constraint, viz. they must be ordered in a ω-sequence. That is to say, all that matters is the structure that the objects possess.

References

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