Abstract: Empirically Feasible Epistemology of Arithmetic

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In this talk we propose a framework for an epistemological theory of arithmetic that takes into account the recent empirical data on proto-arithmetical cognition. As summarized by the neurobiologist Andread Nieder, the state of the art of the empirical research is that:

> Basic numerical competence does not depend on language; it is rooted in biological primitives that can already be found in animals. Animals possess impressive numerical capabilities and are able to nonverbally and approximately grasp the numerical properties of objects and events. Such a numerical estimation system for representing number as language-independent mental magnitudes (analog magnitude system) is thought to be a precursor on which verbal numerical representations build, and its neural foundations can be studied in animal models. (Nieder 2011, 107)

Here we want to pursue the philosophical consequences of such empirical findings. In particular, if the empirical data is correct, how could arithmetical knowledge based on such primitive biological origins be characterized in philosophical terms?

The talk consists of three parts. In the first part, various examples of philosophical relevant empirical data are presented and assessed, drawing from cognitive science, psychology and neurobiology. Based on that data, the best hypothesis about the nature of arithmetical knowledge is that our basic intuitions about small quantities are given by the so-called Approximate number system (ANS), a set of hard-wired "neural filters" responsible for us automatically processing observations in terms of numerosities. However, when we develop the language-based ability to count, we no longer rely on the ANS for our numerical ability. Indeed, even though data show that we never completely lose the ANS, it could be said that the primitive ability is replaced by a language-based one. This gives us the additional expressive power that makes arithmetic as we know it possible.

In the second part, the philosophical relevance and consequences of the empirical data are discussed. The main results are that arithmetical knowledge

thus characterized cannot be purely conceptual under any relevant understanding of conceptual knowledge. The key concept in developing arithmetic into the mathematical discipline we are familiar with, that of *successor,* is at least partly determined by the proto-mathematical intuitions given to us by the ANS. Hence, while arithmetical thinking is underdetermined by its primitive biological origins, it is not independent of them. This way, the best current empirical data suggests an objective basis for arithmetical knowledge, one based on biological primitives.

In the third part, a framework of an empirically feasible epistemological theory is proposed. This theory is characterized as contextual a priori. It takes arithmetical knowledge to be in a context set by the primitive neurological characteristics we share with many animals. However, in that context arithmetical knowledge is essentially a priori as it cannot be corroborated or falsified in the same sense as empirical knowledge. The account of contextual a priori developed here is different from earlier contextual approaches by, e.g., Kuhn (1993) and Putnam (1976). Arithmetical knowledge is neither just another paradigm (against Kuhn) nor can it - if correct - be false (against Putnam). Rather, it is knowledge based on an *inevitable* way of observing the world. As such, arithmetical knowledge is fundamentally objective, and there is a non-conventional way in which arithmetical statements can be true.

Finally, the theory of contextual a priori is argued to fulfil five important requirements of an epistemological theory of arithmetic. First, it contains no unreasonable ontological assumptions. Second, it is epistemologically feasible as a part of a generally empiricist philosophy. Third, it can explain the apparent objectivity of at least some mathematical truths. Fourth, applications of mathematical theories in empirical sciences are not a miracle. Fifth, it does not rid mathematics of its special character. In addition to those five generally accepted criteria, the contextual a priori also fulfils a sixth criterion that we should at this point take seriously in the philosophy of mathematics: it is empirically feasible based on the best data we currently have.

References

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