On the constitutive role of mathematisation in economic modelling

0.1 Summary

In this paper I study one important aspect of mathematisation in economics, namely, the constitutive function of mathematics in model building. I show that this rather crucial aspect of mathematisation has been almost entirely neglected by philosophers of economics working on models and clarify its significance by means of two examples, concerning the representation of preferences and the aggregation of utility respectively. Finally, I show how, by taking seriously the role of mathematics in model-building, one can provide interesting refinements of current accounts of modelling.

0.2 Paper outline

Economic models do not uniformly deal with fully formed empirical concepts, which are later translated into some formal notation for the sake of convenience (the latter view has been advocated in several, very influential papers, notably Samuelson 1952, Gibbard and Varian 1978 and Debreu 1986). The construction of models in mathematical economics constantly calls for active decisions concerning what resources and formal properties have to be employed in order to determine the object of inquiry. In other words, economic objects like commodity spaces, preferences or technologies are ordinarily characterised by means of mathematical determinations (e.g. commodity spaces are subsets of certain real vector spaces, preferences satisfy topological properties spelled out with respect to the latter spaces, technologies are convex). Thus, it would be incorrect to think that there is an array of fully non-mathematical economic concepts to which an independent, parallel mathematical treatment is associated. The concepts in question, insofar they are used in the development of economic theory, are constituted by mathematical conditions or determined within an independently given semantic environment. In this context, mathematics cannot be reduced to a form of mental discipline adopted to ensure the correctness of steps in a piece of reasoning that is otherwise non-mathematical: on the contrary, it is central in concept formation and model building. This amounts to its constitutive function.

In modeling practice, the constitutive function of mathematics is most clearly revealed when the change of semantic environment gives rise to a change in the results that one can establish in economic theory. This is most striking when one looks at problems that depend on a choice of 'canonical' set-theoretical objects, typically the real numbers. A switch to a non-canonical structure, e.g. an elementary extension of the reals containing infinitesimals, may lead to vastly different results. I briefly consider two examples in which this happens: the first concerns the fact that lexicographic orderings are not in general representable on the reals but they are on suitable elementary extensions (this follows from a theorem of Narens 1985), while the second example concerns the fact that infinite utility streams cannot be equitably aggregated on the reals (Basu and Mitra 2003) whereas they can be aggregated in infinitely many distinct ways on an arbitrary enlargement of the reals, i.e., an elementary extension of the type used in nonstandard analysis (this result is implicitly present in Lauwers 2010 and in Pivato 2013, but none of these authors explicitly contrasts the use of the standard and nonstandard reals). These results suggest a way of correcting a recent account of economic models as open formulae (Alexandrova 2008) in a way that makes it substantially more fruitful. The main problem with Alexandrova's account is that she takes an informal, purely linguistic description to be sufficient to capture salient features of modelling. The inadequacy of this approach is revealed by the above examples, in which the search for a representation of a lexicographic ordering or an equitable aggregation is equivalent to the problem of the satisfiability of a certain open formula, in the first-order language of set theory. Now, the satisfiability problem crucially depends on the choice of semantic environment (the reals or an elementary extension). Thus an economic model, if it is to involve open formulae, has to be conceived at least as a complex including a formula and its associated environment or, possibly, an array of environments. By slightly expanding Alexandrova's account into one in which certain classes of models are seen as open formulae plus a semantic environment on which their parameters are interpreted, one obtains a clearer picture of certain decisive choices in economic modeling and even a general strategy to remove negative results from certain branches of economic theory.

References

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