Does Logical Pluralism entail Mathematical Pluralism, or vice versa?

Extended Abstract

In this paper we will consider the question of whether logical pluralism entails mathematical pluralism, and vice versa. For short, we will label this question **LMPQ**. We will suggest the *Austerity Approach* as a general methodology for providing a satisfactory answer to **LMPQ**. Such a satisfactory answer, we argue, should meet the *Stability*-desideratum. We will then move on to consider one particular proposal in reply to **LMPQ**, namely that of Priest (2021), who argues that mathematical pluralism does *not* entail logical pluralism. We will argue that Priest's argument, in light of the *Austerity Approach*, is *unstable*: it needs to have too many theoretical commitments in place in order to work.

Logical pluralism is the thesis that there are multiple different, but equally 'correct' or 'admissible' logics. Mathematical pluralism, on the other hand, starts from the observation that there are various different mathematical practices, where mathematicians develop and study various different mathematical structures using various different foundations and various different techniques. One way to present the issue of plurality, for both logic and mathematics, is to ask whether different such practices and systems are *rivals* to each other, or whether they can co-exist harmoniously and non-rivalrous (Weber 2021). While the pluralist will argue for the latter, a monist would argue for the former.

The issue of whether logical pluralism entails mathematical pluralism, **LMPQ,** has recently been discussed by various authors. For example, Sereni, Fogliani and Zanetti (2021) argue that by endorsing pluralism about abstraction principles in mathematics we obtain a form of logical pluralism. Shapiro (2014) aims to show that as there are different legitimate mathematical practices that rely on classical logic and intuitionistic logic respectively, we have obtained a logical pluralism that considers both classical logic and intuitionistic logic as equally admissible. In that case have obtained logical pluralism because of there being a kind of mathematical pluralism.

Prima facie, an answer to **LMPQ** seems to depend on how one thinks about logical pluralism, mathematical pluralism and the connection between logic and mathematics, respectively. The risk is then that there is no answer to **LMPQ** that is not relative to these variables and **LMPQ** then becomes uninteresting and insignificant. We aim to mitigate this risk of insignificance by adopting a general methodological approach that endorses minimal theoretical commitments for each of the variables involved. We label this the *Austerity Approach*. Our inspiration for this approach is Corcoran (1973), who anticipates much of our current discussion on **LMPQ** by considering gaps between logical theory and mathematical practice. Corcoran recognizes, just as we do, that

> different attitudes and different borders [concerning the respective natures of logic and mathematics] yield different judgements concerning what is and what is not a gap. The most useful attitude/border complex would maximize content and minimize ground for philosophic disagreement (p. 23).

This leads us to propose a particular desideratum for a satisfactory answer to **LMPQ**: *stability*. A *stable* answer is one that can *not* easily be challenged on the basis of the theoretical commitments it requires to take on board in order to work, while an *unstable* answer can.

Priest (2021) argues *against* the conjunction of mathematical pluralism and logical pluralism. However, we argue that his argument is *unstable*. One of the main points of contention for us is Priest's conception of logic (and logical pluralism accordingly). We will here focus on three aspects on the part of his conception of logic that we believe makes his unstable. Priest stipulates that logic has a canonical application, namely that of studying the validity of arguments (in the vernacular), where validity is understood as the preservation of truth *simpliciter*. However, first, the very idea that logic *has* a canonical application has recently been disputed by Commandeur (2022). Second, even if logic has a canonical application, it is not obvious that this would concern *truth*-preservation. As Priest himself notes, ''[o]ne would have to face the fact that in many logics validity is not defined in terms of truth preservation, but in some other terms'' (p. 4944). Third, even if there is a canonical application to logic, and if that canonical application is truth-preservation, it is not obvious that it is truth *simpliciter* that is being preserved; for example, some have proposed that a valid argument merely needs to preserve *some* particular truth property (e.g. Beall 2000). Our point is that on all these three points substantial theoretical commitments need to be endorsed for the argument to work. This goes against the maxim of minimal theoretical commitment on the Austerity Approach that we propose.

In contrast to Priest (2021), we take the proposals of Sereni, Fogliani and Zanetti (2021) and Shapiro (2014) to be more stable. Put briefly, the reason for this is that we take these accounts to better account for the *actual* practice of logic and mathematics, without smuggling in theoretical pre-conceptions about (the nature of) logic. As Sereni, Fogliani and Zanetti (2021) succinctly put it in defense of their proposal:

> A pluralist stance on abstractions […] shares the same merit that Shapiro (2014) praises for logical pluralism, i.e. of presenting itself as the best explanation for the different historical or theoretical role played by more than one notion in the cluster of equally legitimate but still incompatible companions (p. 25).

This, we argue, is more in line with the Austerity Approach that we propose, and thus is more stable. This will lead us to conclude that even in the case that there is strictly speaking no *necessary* connection between logical pluralism and mathematical pluralism, it would be the more natural and *stable* option to endorse logical pluralism if one endorses mathematical pluralism, and vice versa.

References

Beall, J.C. (2000). On mixed inferences and pluralism about truth predicates. *The Philosophical Quarterly*, (50) 200, pp. 380-382.

Commandeur, L. (2022). Against telic monism in logic. *Synthese*, (200) 1, pp. 1-18.

Corcoran, J. (1973). Gaps between logical theory and mathematical practice. In: *The methodological unity of science*, pp. 23-50.

Priest, G. (2021). A note on mathematical pluralism and logical pluralism. *Synthese*, 198 (suppl. 20), 4937-4946).

Sereni, A, Fogliani, M. P. S, and Zanetti L. (2021). For better and for worse: abstractionism, good company, and pluralism. *The Review of Symbolic Logic*, pp. 1 – 30.

Shapiro, S. (2014). *Varieties of Logic*. Oxford University Press: Oxford.

Weber, Z. (2021). Relevant arithmetic and mathematical pluralism. *The Australasian Journal of Logic*, 18 (5), pp. 569-596.